23rd International Congress on Sound & Vibration
Athens, Greece 10-14 July 2016
ICSV23

THE RELATIONSHIP BETWEEN FLOOR VIBRATION FROM AN UNDERGROUND SOURCE AND AIRBORNE SOUND PRESSURE LEVEL IN THE ROOM

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Prediction, measurement and assessment of groundborne noise in buildings due to the operation of underground railways frequently involves knowledge of floor vibration and requires a calculation to be made to estimate the airborne sound pressure level that will be caused by the floor vibration. A rule-of-thumb conversion between vertical velocity level and airborne sound level is frequently made. Approximations using calculations of radiation efficiency of the floor plate and the room reverberation time are also sometimes made. This paper reviews the classical equations for coupling between finite plates and rectangular rooms using wave acoustics, and their application to floor plates in buildings at low frequencies. It also compares calculated relationships between floor velocity and room sound pressure level with numerically modelled sound pressure in the airspace above a vibrating plate. Conclusions are drawn regarding appropriate ways of assessing airborne noise caused by a vibrating floor plate when only the floor vibration is known.

1. Introduction

In the field of groundborne noise from underground railways, it is frequently necessary to predict the relationship between the vibration of a floor plate (or other surface) and the sound pressure level in a room above the floor, or into which the vibration of the surface causes radiation of sound. Kurweil [1] reviewed the topic and reported that a reasonable estimate could be made of the relation between the octave-band room floor acceleration level, and the resulting octave band sound pressure level in the room. When expressed in terms of velocity level, Kurzweil's equation simplifies to

$$L_p=L_v-27 \text{ dB} \tag{1}$$

Where L_v = floor vertical velocity level in dB re 1 nanometre/second and L_p = sound pressure level in the room in dB re 20 µPa. This relationship has been widely used, including a similar version used in the USA that $L_p=L_v$ when L_v is in dB re 1 microinch/second, equivalent to $L_p=L_v$-28 dB, although Zapfe et al. [2] reported that a better model was that $L_{pAwt}=L_{vAwt}$-33 dB where the "Awt" subscript signifies the overall A-weighted level. However, they presented a group of measured results plotted against this equation which showed some measurements over 10 dB higher, with the highest being 15 dB higher.

A more detailed approach is to calculate the relationship using wave acoustics, or to model the system numerically. The purpose of this paper is to review the underlying principles and compare predictions made using the range of methods outlined above.

2. Underlying principles

2.1 Sound radiated by a plate into a rectangular room

The sound pressure level which occurs in a room, one surface of which is a vibrating plate, is a function of the plate mode shape function and the room mode shape function. The coupling between them is determined by the product of the two functions integrated over the surface of the plate. Let us consider a room with dimensions L_x, L_y and L_z, where L_z is the height of the room and one surface in the *x,y* plane, i.e. the floor, is vibrating.

For a simply supported plate the shape function is [3]

$$\psi(x,y) = sin\frac{m\pi x}{L_x}\ sin\frac{n\pi y}{L_y} \tag{2}$$

where *m* and *n* are any integer greater than 0. This is associated with plate eigenfrequencies which are given by

$$\omega_N = \sqrt{\frac{D}{\rho_s}}\left[\left(\frac{m\pi}{L_y}\right)^2 + \left(\frac{n\pi}{L_x}\right)^2\right] \tag{3}$$

Where $D=\frac{Eh^3}{12(1-\sigma^2)}$ is the bending stiffness, ρ_s is the surface density, σ is Poisson's ratio, *h* is the plate thickness and *E* is Young's modulus.

Mode 1,1 is the lowest plate mode with an antinode in the centre of the plate, although mode 0,0 can exist if the plate is moving vertically as a piston, with consequences discussed below. If a floor plate is vibrating as a piston it will have increased coupling with the air in the room, but only at room mode 0,0,*r*. In the case of clamped edges, the frequency of the lowest eigenmode will be much higher than in the case of simply supported edges, often above the frequency range in which groundborne noise from underground railways has its peak.

The room mode shape function is [4]

$$\phi(x,y,z) = cos\frac{p\pi x}{L_x}\ cos\frac{q\pi y}{L_y}\ cos\frac{r\pi z}{L_z} \tag{4}$$

where *p*, *q* and *r* are any integer including 0.

This is associated with the room eigenfrequencies which for a lossless room are given by

$$\omega_N = \pi c\sqrt{\left(\frac{p}{L_x}\right)^2 + \left(\frac{q}{L_y}\right)^2 + \left(\frac{r}{L_z}\right)^2} \tag{5}$$

Mode 0,0,0 occurs when the air in the room is being compressed, but the forcing frequency of the compression is too low, and therefore the wavelength too long, to cause a standing wave. The eigen-frequency for mode 0,0,0 is zero. Given that the floor plate has a finite mass, in reality mode 0,0,0 is the mass-air resonance of the floor plate as a mass on an air spring, i.e. $\omega_{0,0,0} = \sqrt{\left(\frac{L_x L_y \rho c^2}{L_z \rho_s}\right)}$

Equations 2 and 4 are widely used in airborne acoustics in considering coupling between walls and rooms although other functions are required for other edge conditions.

Coupling between the plate and the room is given by integrating the product of the plate shape function and the room shape function for z=0 where z is the room height if the plate is the floor. This integral leads to the result that no coupling occurs when m+p or n+q is even. This, however, does not take into account the fact that in the case of room heights which are less than the other room dimensions, which is usually the case, the measurement point is in the near field of the floor plate. The reason coupling does not occur in the far field is that adjacent zones of the plate are in antiphase, while the room mode shape is symmetrical, so sound radiated from one zone is cancelled by the antiphase radiation of the adjacent zone. In the near field, however, this cancelling involves volume flow along the path between the two zones, and resulting finite sound pressure.

Ignoring the near-field effect, the sound pressure at a point x,y,x in the room, for a shape function for a simply supported plate, is then given by [6][7]

$$\frac{p_{x,y,z}}{v_{x,y}} = \frac{i\omega\rho c_o^2}{\pi^2 L_z}\left[\sum_{p,q,r=0}^{\infty} \frac{\Lambda_{p,q,r}(-1)^r \phi(x,y,z)\left[\sum_{m,n=1}^{\infty} \frac{mn[(-1)^{m+p}-1][\left(-1\right)^{n+q}-1]}{(p^2-m^2)(q^2-n^2)}\right]}{[\left(\omega_{p,q,r}+i\delta_{p,q,r}\right)^2-\omega^2]\psi(x,y)}\right] \tag{6}$$

Where

$$\Lambda_{p,q,r} = \varepsilon_p \ \varepsilon_q \ \varepsilon_r \tag{7}$$

and $\varepsilon_p = 1$ for $p = 0$; $\varepsilon_p = 2$ for $p > 0$; $\varepsilon_q = 1$ for $q = 0$; $\varepsilon_q = 2$ for $q > 0$; $\varepsilon_r = 1$ for $r = 0$; $\varepsilon_r = 2$ for $r > 0$;

The damping term δ is related to the reverberation time

$$\delta =\cong \frac{6.9}{T} \tag{8}$$

and the reverberation time is given by

$$T \cong \frac{0.161}{\frac{\varepsilon_p\alpha_{y,z}}{L_x}+\frac{\varepsilon_q\alpha_{x,z}}{L_y}+\frac{\varepsilon_r\alpha_{x,y}}{L_z}} \tag{9}$$

where $\alpha_{y,z}$, $\alpha_{x,z}$, $\alpha_{x,y}$ are the average absorption coefficients of the room surfaces in the y,z, x,z and x,y planes. It should be noted that for constant α the reverberation time is dependent on the mode number.

Where the measurement height $z<\frac{L_z}{m}$, m+p in the numerator should be replaced by 1 and where $z<\frac{Ly_z}{n}$, n+q in the numerator should be replaced by 1. When m=p or n=q, the integral takes a different form, and m+p or n+q should be set to zero, avoiding the singularity.

These approximations are only valid for small values of α which is likely to be the case at low frequencies in most rooms. In such cases, wall absorption is not due to a lining of soft material, but due to transmission of sound through the wall. This is a function of the admittance ratio of the wall [5].

In the case of sound pressure in a room due to vibration from an underground railway, it is usual to measure both the floor vibration and the room sound pressure level at discrete points, not necessarily with the same x and y co-ordinates. It is clear from equation 2 that the measured amplitude of the floor plate vibration will be dependent in the position of the measuring point, being at its maximum at an antinode. Likewise the sound pressure will be at its maximum in the corners of the room and minimum at a node within the room. It is thus sensitive to both position on the floor and height above the floor. It is a common practice to measure ground-borne sound levels in rooms near the centre of the room, and this may result in a level being measured lower than the maximum which occurs in the corners. The position on the floor plate where the vibration amplitude is measured may or may not be at an antinode.

The results obtained with equation 6 have been compared with the results obtained using a finite difference method as implemented by the *FINDWAVE* program [8]. For this purpose the room was assumed to have near zero absorption on all walls except the floor, where the value of $\alpha_{x,y}$ was set as 4κ where κ the admittance ratio of the plate [5]. Figure 1 shows $20\ log_{10}\left|\frac{p_{x,y,z}}{v_{x,y}}\right|$ for a measurement location for both v and p in which x,y and z are just over 40% of the room dimension. L_x is 3.5m, L_y is 4m and L_z is 3m. The results in Figure 1 have been evaluated with using equation 6 after taking account of the change of reference level from 1 nm/s to 20 µPa, requiring the subtraction of 86 dB, over the frequency range 0-140Hz within which groundborne noise usually occurs.

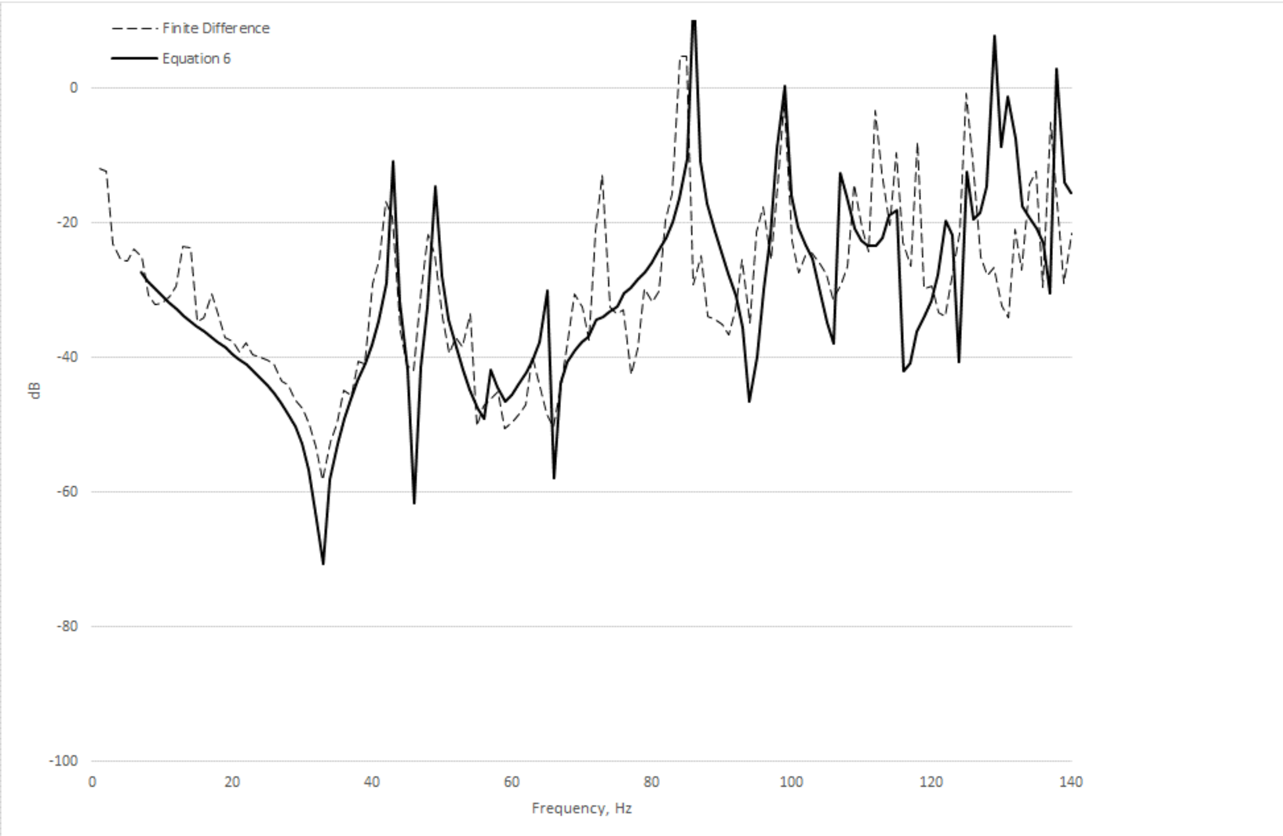

Figure 1: L_p-L_v for a near lossless room 3.5m x 4m x 3m high

An important feature of the results is the presence of a series of anti-resonances, and the prominence of the widely spaced room modes between 40 and 100Hz.

In the case of a thick concrete slab, the lowest slab mode may be above the peak frequency in the incoming groundborne noise spectrum, and the slab may be in forced vibration at mode 0,0, i.e. the entire building may be vibrating vertically as a piston. In this case the wall coupled with the room at mode 0,0,1 with a greater efficiency by a factor of $\frac{\pi^2}{4}$ or 7.8 dB. That will mean the walls are also in vertical vibration in phase with the floor, and the slab which forms the ceiling of the room, if of similar construction, will be vibrating in phase with the floor slab. In so doing it will be radiating downwards a similar pressure field in antiphase to that radiated by the floor, and the room shape function becomes

$$\phi(x,y,z) = cos\frac{p\pi x}{L_x}\, cos\frac{q\pi y}{L_y}\left\{cos\frac{r\pi z}{L_z} - cos\frac{r\pi(L_z - z)}{L_z}\right\} \tag{10}$$

This shape function would also occur in a tall building with similar floors.

The reverberation time associated with the assumptions in the computation for Figure 1 is very long, and in practice, rooms in which measurements are made have much shorter reverberation times, and in building acoustics it is common to normalise room sound measurements to a reverberation time of 0.5 second. Figure 2 shows the results for the same dimensions as figure 1 but with a reverberation time of 0.5 second at all frequencies.

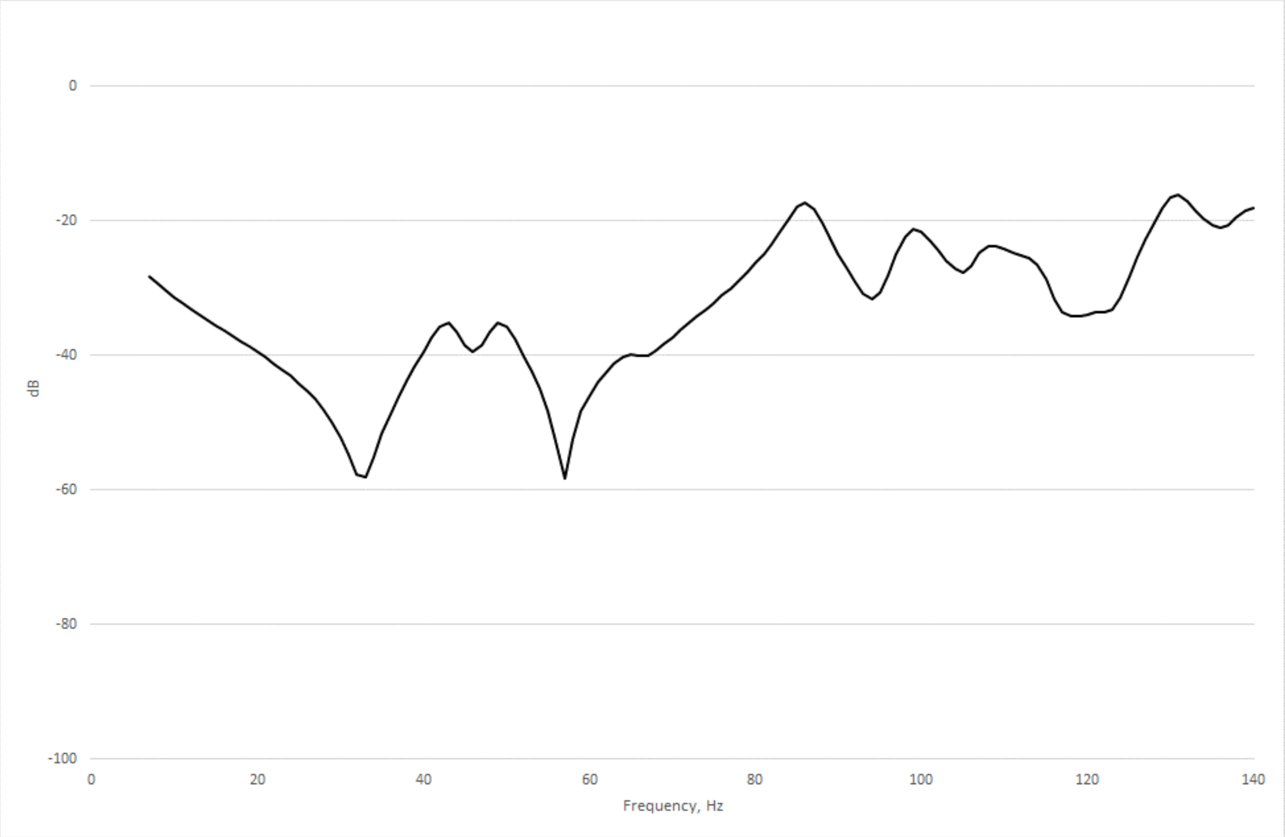

Figure 2: L_p-L_v for a room 3.5m x 4m x 3m high with reverberation time of 0.5s.

It can be seen that the relationship between sound pressure and velocity levels will strongly depend on the relationship between the frequencies of plate modes in the floor and those of room modes. If a plate mode should correspond with a room mode $L_v - L_p$ may be less than 20 dB, but if it falls at the frequency of an antiresonance it is very much greater. Depending on the approach taken to uncertainty, one may either take the worst case or alternately take a probability approach and use the mean square in third octave bands, as in Figure 3.

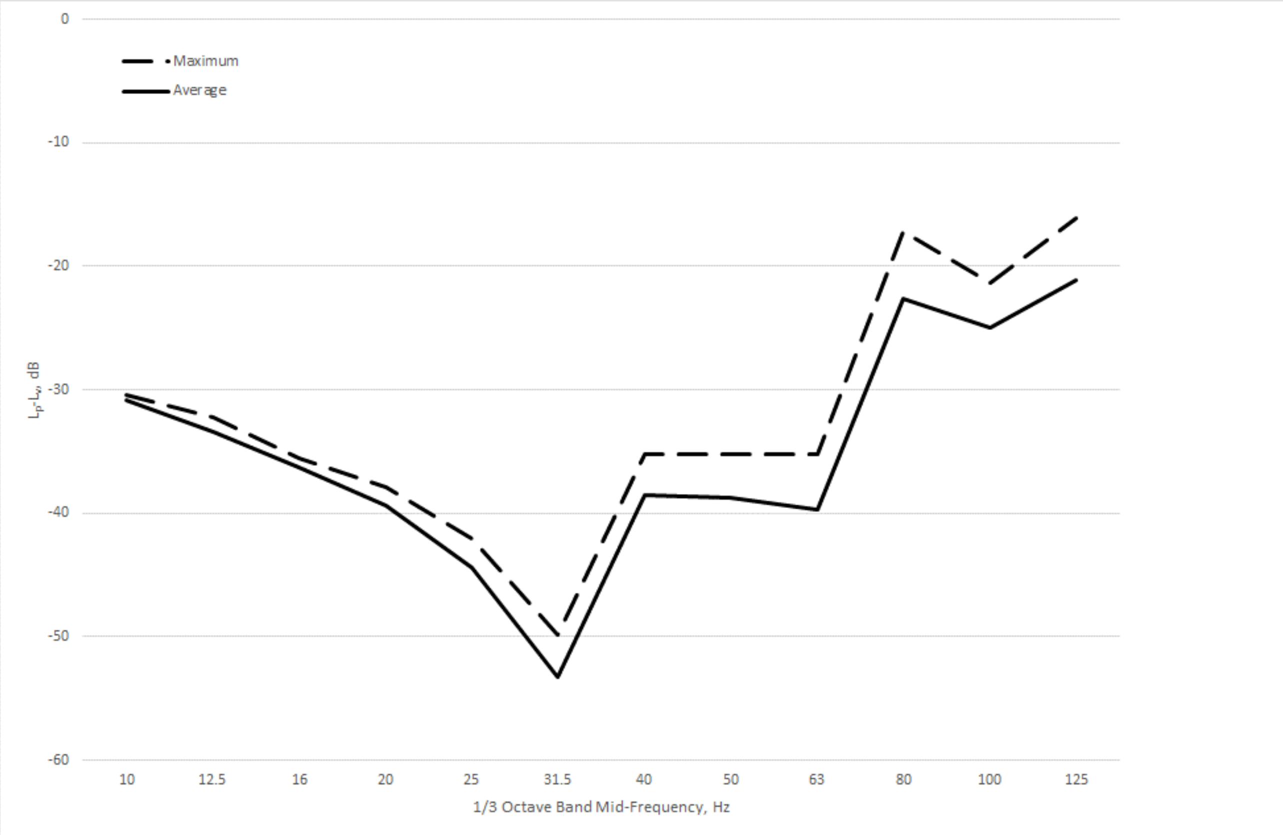

Figure 3: Maximum and mean square average of L_p-L_v in third octave bands for room 3.5m x 4m x 3m high with reverberation time of 0.5s.

These results may be adjusted for a reverberation time *T* other than 0.5s by adding the correction $20\,log_{10}\frac{T}{0.5}$. This is a worst case applying strictly only to frequencies of room modes. The results may be adjusted for other locations, if the floor plate can be assumed to be simply supported, in the room using the correction

$$20\,log_{10}\frac{sin\frac{m\pi x}{L_x}\,sin\frac{n\pi y}{L_y}}{cos\frac{p\pi x}{L_x}\,cos\frac{q\pi y}{L_y}\,cos\frac{r\pi z}{L_z}} \tag{11}$$

where *x* and *y* in the numerator are the vibration measuring position s and *x*, *y* and *z* in the denominator are the sound measuring position.

Figures 4 and 5 show corresponding results for a selection of other room sizes.

3. Conclusions

It can be shown that the relationship between floor velocity and room sound pressure level varies widely according to (i) the relationship between measurement locations of the two quantities, (ii) the room dimensions (iii) the reverberation time of the room and, most importantly, the relationship between the frequencies at which plate modes in the floor occur and airborne room modes. If, as is often the case with groundborne noise from underground railways, the floor vibration is concentrated at a single or small number of modes, slight differences between the match/mismatch between floor plate modes and airborne room modes may change the result by the order of 10 dB. In a case where it is essential not to underestimate the sound pressure level in a room likely to result from known or predicted floor vibration, a worst case approach should be taken in which the modes are assumed to coincide. Otherwise a 1/3 octave band average may be used. Equation 6 can be implemented numerically in a spreadsheet.

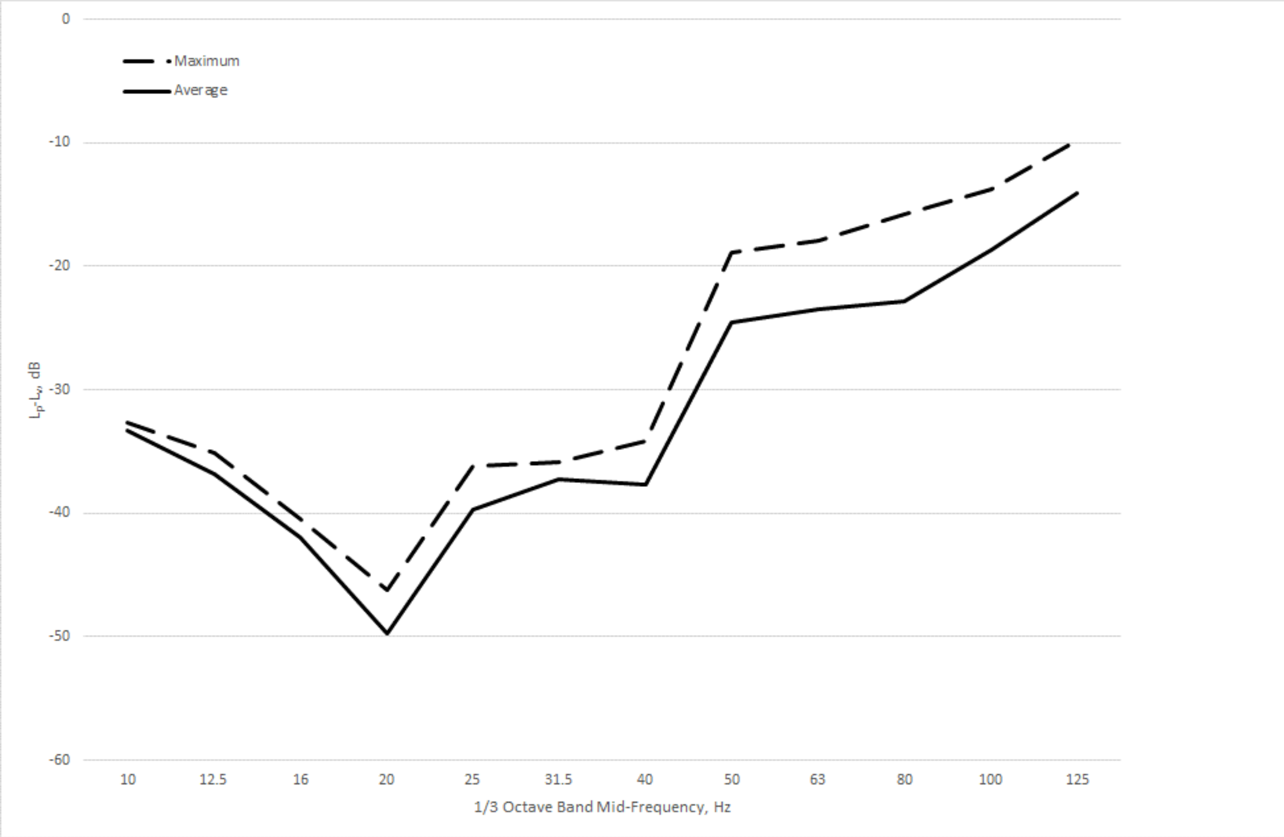

Figure 4: Maximum and mean square average of L_p-L_v in third octave bands for room 6m x 5m x 3m high with reverberation time of 0.5s.

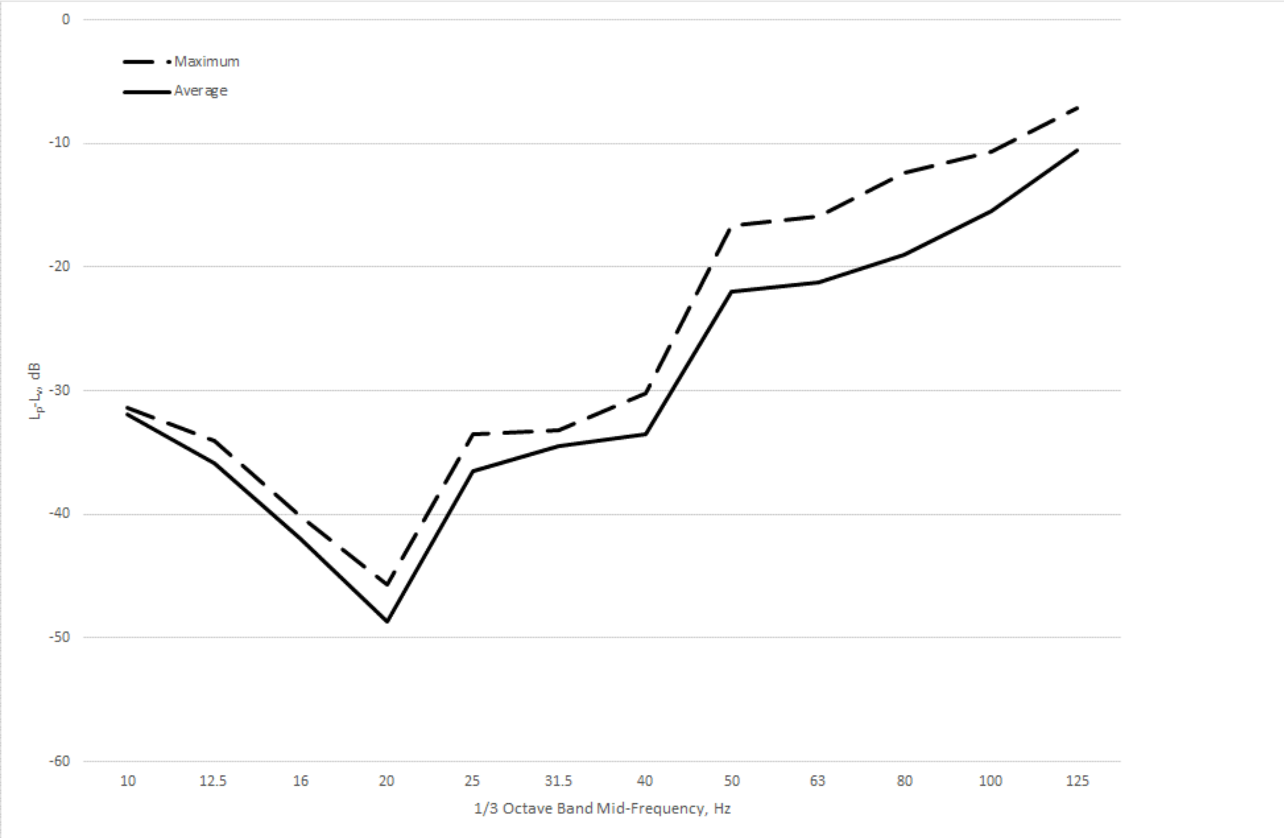

Figure 5: Maximum and mean square average of L_p-L_v in third octave bands for room 6m x 5m x 2.5m high with reverberation time of 0.5s.

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